JHEP HYPER VERSION

Received: *February 10, 1998,* Accepted: *February 17, 1998*

Black holes and elementary string states in $N = 2$ supersymmetric string theories

Ashoke Sen
Mehta Research Institute of Mathematics and Mathematical Physics
Chhatnag Road, Jhoosi, Allahabad 211019, India
E-mail: sen@mri.ernet.in

Abstract: We compare the logarithm of the degeneracy of BPS saturated elementary string states and the string modified Bekenstein-Hawking entropy of the corresponding black holes in $N = 2$ supersymmetric heterotic string compactification to four dimensions. As in the case of $N = 4$ supersymmetric theory, the two results match up to an overall undetermined numerical factor. We also show that this undetermined numerical constant is identical in the $N = 2$ and $N = 4$ supersymmetric theories, therby showing that the agreement between the Bekenstein-Hawking entropy and the microscopic entropy for $N = 2$ theories does not require any new identity, other than the one already required for the $N = 4$ theory. A similar result holds for type II string compactification as well.

Keywords: Black Holes in String Theory, Superstrings and Heterotic Strings.

JHEP02(1998)011

Contents

1 Introduction

The idea of relating black holes to elementary string states is quite old [1, 2, 3, 4]. One particularly fascinating similarity between black holes and the elementary string states is the large amount of degeneracy of states of each of them. For black holes the existence of this large degeneracy is deduced indirectly from the large Bekenstein-Hawking entropy carried by a macroscopic black hole. On the other hand, for elementary string states, this large degeneracy is a consequence of the Hagedorn spectrum of states in string theory. Despite this qualitative similarity, the naive attempt to compare the entropy of a Schwarzschild black hole to the logarithm of the degeneracy of elementary string states runs into difficulty due to the fact that the former is proportional to the square of the mass of the black hole, whereas the later is proportional to the mass of the elementary string states. It was suggested in [2] that this discrepancy is due to the large mass renormalization suffered by the state due to quantum corrections. This idea was further developed in [5, 6, 7, 8]. Other attempts at providing a microscopic explanation of the entropy of a Schwarzschild black hole have been made in [9, 10].

In supersymmetric string theories, there are a class of states, known as BPS states, which do not suffer any mass renormalization, and hence can avoid the difficulty mentioned above [11]. This feature was exploited in [12] to compare the entropy of extremal electrically charged BPS black holes in heterotic string theory compactified on T^6 with the logarithm of the degeneracy of elementary string states carrying the same charge. In this case, however, the difficulty arises from that fact that the extremal electrically charged black hole has vanishing area of the event horizon, and is singular at the horizon. This leads to an apparent contradiction since the microscopic calculation based on the degeneracy of the elementary string states gives a non-vanishing answer for the entropy. However, one should keep in mind that the black hole solution that has vanishing area of the event horizon is obtained by solving the low energy equations of motion of string theory. These equations are valid far away from the horizon, but are expected to be modified near the horizon where the curvature is large. Since for these black holes the string coupling constant goes to zero near the horizon, the relevant corrections to be taken into account are the world-sheet σ-model corrections *but not string loop corrections.* Although our present technology does not allow us to calculate this correction explicitly, a general scaling argument was used in [12] to determine the modified area of the event horizon up to an overall numerical factor, which could not be determined from the scaling argument. The result agreed exactly with the microscopic entropy calculated from the degeneracy of elementary string states up to this overall numerical factor. This calculation was later generalized to toroidally compactified heterotic string theory in higher dimensions [13]. Further support to the identification of extremal black holes with elementary string states was provided in [14, 15, 16, 17, 18] by comparing the scattering involving these two sets of states. Refs. [15, 19] explicitly identified the states responsible for black hole degeneracy by constructing classical solutions describing oscillations of the underlying microscopic string. A somewhat different approach to this problem has been discussed in [20].

Since then great progress has been made towards understanding the microscopic origin of black hole entropy by comparing the Bekenstein-Hawking entropy to the microscopic entropy of appropriate configuration of D-branes [21] carrying the same charge as the black hole [22]. The main advantage of using D-branes instead of elementary string states is that for an appropriate configuration of D-branes, the event horizon of the corresponding black hole is non-singular and has finite area. Thus the Bekenstein-Hawking entropy for these black holes can be calculated unambiguously, and can be compared with the corresponding microscopic answer obtained from the counting of states of the D-brane. The two calculations turn out to be in exact agreement, including the overall numerical factor. Since then this calculation has been generalized to many other classes of black holes [23], including black holes in $N = 2$ supersymmetric string theory [24, 25, 26, 27, 28, 29, 30, 31, 32, 33]. Much progress has also been made in extending this analysis to slightly non-extremal black holes [34] and in providing a microscopic explanation of Hawking radiation from these black holes [35, 36, 37]. An alternative derivation of the microscopic entropy of these black holes has also been attempted [38, 39, 40] by mapping this problem to that of counting of elementary string states.

Although these developments mark tremendous progress in our understanding of microscopic description of black holes, they do not directly address the problem of studying the relationship between elementary string states and black holes. The results of [12, 13] provide strong evidence for such a relationship, since here an explanation for the Bekenstein-Hawking entropy of the black hole is provided in terms of the degeneracy of the microscopic states of the elementary string excitations. In particular, if one could take into account the higher derivative terms in the string effective action to compute the overall numerical constant in the expression for the Bekenstein-Hawking entropy and show that the result agrees with the microscopic entropy computed from the degeneracy of string states, it would put the correspondence on a much firmer ground. However, as we have already mentioned, our current technology in string theory does not permit us to do this computation. Another direction could be in extending these results to non-supersymmetric black holes. Some progress in this direction has been made in

[41, 42, 43, 44, 45].

In this paper we shall generalise the result of [12] to more general string theories in four dimensions. Since for the comparison to be meaningful we need BPS states, we need the four dimensional theory to have at least $N = 2$ supersymmetry. We shall focus our attention on the most general $N = 2$ string compactification[1], but as we shall see, based on this analysis we shall also be able to make some general remark about more general $N = 4$ string theories other than the ones obtained by toroidal compactification of heterotic string theory [47].

There are several ways to get $N = 2$ supersymmetric string theories in four dimensions. One class of these theories is obtained by heterotic string compactification on a conformal field theory with (0,4) world-sheet supersymmetry. The simplest example is heterotic string theory on $K3 \times T^2$, but more general compactifications based on asymmetric orbifolds, and other exotic conformal field theories are possible [48, 49]. This is the class of theories on which we shall focus most of our attention. We can also get $N = 2$ supersymmetric string theories from compactifying type II string theory on Calabi-Yau manifolds. But in these theories all gauge fields arise from the Ramond-Ramond sector, and as a result, none of the elementary string states are BPS states since they do not carry any Ramond-Ramond charge. Finally, we can also get $N = 2$ supersymmetric string theories from asymmetric compactification of type II theories where all the space-time supersymmetries come from the right (or left) moving sector on the world-sheet [50]. These theories do have BPS saturated elementary string states. As we shall see, the analysis for these theories is very similar to that for the heterotic string theories.[2]

We shall show that for the most general $N = 2$ heterotic string compactification, the Bekenstein-Hawking entropy of an extremal electrically charged black hole agrees with the microscopic entropy calculated from the degeneracy of elementary string states up to an overall undetermined numerical factor, in a manner identical to that in the case of toroidally compactified heterotic string theory. More importantly, we shall show that the computation of this overall numerical factor in the $N = 2$ theories is independent of the details of the compactification and is identical to that in the case of toroidally compactified heterotic string theory. In other words if one is able to compute this factor for the toroidally compactified heterotic string theory and show precise agreement between the Bekenstein-Hawking entropy and the microscopic entropy for this theory, then it also guarantees precise agreement bewtween the two entropies in all $N = 2$ supersymmetric heterotic compactification. The same result holds for non-toroidal $N = 4$ supersymmetric heterotic string compactification discussed in [51].

A similar story is repeated for type II string compactification. For any four dimen-

[1] Although for $N = 2$ supersymmetric theories the degeneracy of string states can jump discontinuously as we move in the moduli space [46], we shall work at weak string coupling where such phenomena are not expected to occur.

[2] One could also consider compactified type I theory, but the spectrum of elementary string states in this theory only has a limited number of BPS states associated with states carrying internal momentum. In particular, there are no BPS elementary string states with large degeneracy.

JHEP02(1998)011

sional type II string compactification with at least two supersymmetries coming froim the right-moving sector of the world-sheet, – toroidal compactification with $N = 8$ supersymmetry, symmetric or asymmetric compactification with $N = 4$ supersymmetry, or asymmetric $N = 2$ supersymmetric compactification – the microscopic entropy associated with elementary BPS saturated type II string states agrees with the Bekenstein-Hawking entropy of the corresponding extremal black hole up to an overall numerical constant. This constant is again identical in all of these type II string compactifications. However *it is different from the corresponding constant in the heterotic string compactification.* This is just as well, since the required value of this constant for exact agreement between the two entropies is different in type II and heterotic string theories, due to different pattern of growth of the number of states with energy in the two theories.

2 Black hole entropy

At tree level, the massless bosonic field content in the most general heterotic string compactification to four dimensions with $N = 2$ supersymmetry at a generic point in the moduli space is the metric $g_{\mu\nu}$, $(r+2)$ U(1) gauge fields $A_\mu^{(a)}$ $(0 \le \mu \le 3,\ 1 \le a \le (r+2))$ and $2r + 2$ scalars locally parametrizing the moduli space [52, 53, 54, 55, 56, 57]:

$$\frac{SO(2,r)}{SO(2) \times SO(r)} \times \frac{SL(2,R)}{U(1)} . \tag{2.1}$$

The scalars labelling the first coset may be represented by an $(r+2) \times (r+2)$ matrix M satisfying,

$$M^T = M, \qquad MLM^T = L , \tag{2.2}$$

where

$$L = \begin{pmatrix} I_2 & \\ & -I_r \end{pmatrix} . \tag{2.3}$$

I_n denotes $n \times n$ identity matrix. The second coset is labelled by a complex scalar λ taking value in the upper half plane:

$$\lambda \equiv \lambda_1 + i\lambda_2 = a + ie^{-\Phi} , \tag{2.4}$$

where a is the axion field obtained by dualizing the rank two antisymmetric tensor field, and Φ is the dilaton field. This moduli space gets modified by quantum corrections [55, 56], but since the string coupling vanishes at the black hole horizon, we shall not need to consider these corrections. There may also be massless scalars belonging to the hypermultiplet but they will play no role in our discussion, since in constructing the black hole solution we shall set these hypermultiplet fields to zero. The tree level low energy effective action involving the metric and the vector multiplet fields $A_\mu^{(a)}$, M and λ is given by

$$\begin{aligned} S &= \frac{1}{32\pi} \int d^4x \sqrt{-g} \Big[R - g^{\mu\nu} \frac{\partial_\mu \lambda \partial_\nu \bar{\lambda}}{2(\lambda_2)^2} + \frac{1}{8} g^{\mu\nu} Tr(\partial_\mu ML \partial_\nu ML) - \\ &\quad - \lambda_2 g^{\mu\mu'} g^{\nu\nu'} F^{(a)}_{\mu\nu} (LML)_{ab} F^{(b)}_{\mu'\nu'} + \frac{1}{2} \lambda_1 (\sqrt{-g})^{-1} \, \epsilon^{\mu\nu\rho\sigma} F^{(a)}_{\mu\nu} L_{ab} F^{(b)}_{\rho\sigma} \Big] , \end{aligned} \tag{2.5}$$

where $F^{(a)}_{\mu\nu}$ is the field strength associated with $A^{(a)}_\mu$ and R is the Ricci scalar. Our normalization conventions are identical to those in [12, 58]. This action is invariant under the $SO(2,r)$ transformation:

$$M \to \Omega M \Omega^T, \quad A^{(a)}_\mu \to \Omega_{ab} A^{(b)}_\mu, \quad \lambda \to \lambda, \quad g_{\mu\nu} \to g_{\mu\nu}\,, \tag{2.6}$$

where Ω is an $(r+2)\times(r+2)$ matrix satisfying

$$\Omega L \Omega^T = L\,. \tag{2.7}$$

The equations of motion are also invariant under an $SL(2,R)$ transformation [58].

The form of the action (2.5) is dictated to a large extent by the local $N=2$ supersymmetry of the theory [57]. Note that this action is the restriction of the $SO(6,22)$ invariant effective action [59, 58] for heterotic string theory on T^6 to a subspace where the $SO(6,22)$ matrix M is restricted to take value in an $SO(2,r)$ subgroup, and the 28 gauge fields in the vector representation of $SO(6,22)$ are restricted to lie in the subspace transforming in the vector representation of $SO(2,r)$. However, there is a much more powerful result relating these two effective actions that we shall need. In general both, the action of the toroidally compactified heterotic theory and that of the $N=2$ supersymmetric theory, will receive higher derivative corrections even at the string tree level. These can be viewed as coming from higher loop contribution to the β-function of the corresponding σ-model [60, 61], or, equivalently, from order α' and higher order corrections to the tree level string S-matrix elements. It turns out that even after these higher derivative terms are included, the *tree level* effective action involving the metric and the vector multiplet fields of the $N=2$ supersymmetric heterotic string theory is given by the restriction of the tree level effective action of the corrsponding toroidally compactified heterotic string theory.

This can be seen as follows. The $(r+2)$ vector fields in the $N=2$ theory are associated with two right-moving $U(1)$ super-currents $(j^\alpha(z), J^\alpha(z))$ $(1\le\alpha\le 2)$ and r left moving $U(1)$ currents $\bar J^i(\bar z)$ $(3\le i\le (r+2))$.[3] Here j^α, J^α and $\bar J^i$ have dimensions $(0,\frac{1}{2})$, $(0,1)$ and $(1,0)$ respectively. Let X^μ and ψ^μ denote respectively the bosonic coordinates and their right-handed superpartners associated with the non-compact directions. Then the vertex operators of the $(r+2)$ vector fields in the -1 representation [62, 63] are given by [49, 64, 55]:

$$\begin{aligned} V^{\alpha\mu}_{-1} &= e^{-\phi} j^\alpha(z)\bar\partial X^\mu(\bar z) e^{ik\cdot X}\,, \\ V^{i\mu}_{-1} &= e^{-\phi}\psi^\mu(z)\bar J^i(\bar z) e^{ik\cdot X}\,, \end{aligned} \tag{2.8}$$

where ϕ is the bosonized super-ghost. The same vertex operators in the 0 representation are given by

$$\begin{aligned} V^{\alpha\mu}_{0} &= J^\alpha(z)\bar\partial X^\mu(\bar z) e^{ik\cdot X}\,, \\ V^{i\mu}_{0} &= \Big(\partial X^\mu(z) + ik\cdot\psi(z)\psi^\mu(z)\Big)\bar J^i(\bar z) e^{ik\cdot X}. \end{aligned} \tag{2.9}$$

[3] In our convention the world-sheet and hence space-time supersymmetry is in the right-hand sector. Also the right-handed currents are holomorphic.

The vertex operators of the $2r$ scalars parametrizing the coset $SO(2,r)/(SO(2)\times SO(r))$ in the -1 picture are:

$$V_{-1}^{\alpha i} = e^{-\phi} j^\alpha(z) \bar{J}^i(\bar{z}) e^{ik\cdot X} . \tag{2.10}$$

The same vertex operators in the 0 picture are:

$$V_0^{\alpha i} = \Big(J^\alpha(z) + ik\cdot\psi(z) j^\alpha(z) \Big) \bar{J}^i(\bar{z}) e^{ik\cdot X} . \tag{2.11}$$

Finally the vertex operators for the metric $g_{\mu\nu}$ and the scalars λ in both -1 and 0 picture are constructed solely in terms of the world-sheet fields X^μ, ψ^μ and their derivatives, and the (super-)ghost fields.

The tree level S-matrix involving the metric and the particles in the vector multiplet are computed from the correlation functions of various combinations of these vertex operators on the sphere. As can be seen from the structure of the vertex operators, these correlation functions are completely independent of the details of the conformal field theory describing this compactification. The correlators of operators constructed from X and ψ of course are manifestly independent of this conformal field theory. But on the sphere, even the correlators of j^α, J^α and $\bar{J}^i$ are determined solely from the $U(1)$ (super-) current algebra and hence are insensitive to the details of the conformal field theory. Note that this argument does not assume that these (super-) currents are decoupled from the rest of the conformal field theory. In particular there may be non-trivial correlation between the U(1) charges carried by a state and the vertex operator representing it in the rest of the conformal field theory. These states affect the correlation function of the currents on the torus and the higher genus Riemann surfaces, but not on the sphere.

Note in particular that vertex operators with identical structure and correlation functions exist in toroidally compactified heterotic string theory as well. In heterotic string theory on T^6, we can take the right-moving $U(1)$ supercurrents (j^α, J^α) to be two of the six right-moving supercurrents, and the left-moving $U(1)$ currents $\bar{J}^i$ to be the r of the 22 left-moving $U(1)$ currents in this world-sheet theory. The S-matrix elements and hence the effective action involving these restricted set of fields will be insensitive to the fact that we now have a toroidal compactification instead of $N=2$ supersymmetric compactification, and hence will yield the same answer. This shows that in an $N=2$ supersymmetric heterotic compactification, the tree level S-matrix, and hence the effective action involving the metric and the vector multiplet fields is independent of the choice of the internal conformal field theory, and is given by an appropriate restriction of the full tree level effective action of the toroidally compactified heterotic string theory.

Given this result, we proceed as follows. Since the effective action of the $N=2$ theory is the restriction of the effective action of the $N=4$ theory, the electrically charged black hole solutions in the $N=2$ theory are simply the corresponding black hole solutions in the $N=4$ theory with appropriate restriction on the charge. In this case since there are $(2+r)$ $U(1)$ gauge fields, we have a $(2+r)$ dimensional charge vector $Q^{(a)}$. As in [12] we define the left and the right-handed components of the charge as:

$$Q_R^2 = \frac{1}{2} Q^{(a)} (L\langle M\rangle L + L)_{ab} Q^{(b)} ,$$

$$Q_L^2 \quad = \quad \frac{1}{2} Q^{(a)} (L \langle M \rangle L - L)_{ab} Q^{(b)} \,, \tag{2.12}$$

where $\langle\ \rangle$ denotes the asymptotic value of a field in the presence of a black hole. The BPS condition requires[4]:

$$m^2 = Q_R^2 / 8g^2 \,, \tag{2.13}$$

where m is the ADM mass of the black hole, and g is the string coupling constant. Arguments identical to that in ref. [12] give the following expression for the string modified Bekenstein-Hawking entropy of the black hole:

$$S_{BH} = \frac{2\pi C}{g} \sqrt{m^2 - \frac{Q_L^2}{8g^2}} \,. \tag{2.14}$$

Here C is the undermined numerical constant alluded to before. The computation of C involves the higher derivative terms in the effective action *at the string tree level*, but not string loop corrections, since the string coupling constant vanishes as we approach the horizon. Since these higher derivative terms are identical to the ones in the toroidally compactified heterotic string theory, we conclude that the numerical constant C must also be identical to that in heterotic string theory on T^6.

We now need to compute the microscopic entropy by counting the degeneracy of the BPS saturated elementary string states with the same quantum number. In the normalization convention used in [58, 12] the NS sector mass formula for an elementary string states carrying charge Q is given by:

$$m^2 = \frac{Q_R^2}{8g^2} = \frac{g^2}{8} \Big(\frac{Q_L^2}{g^4} + 2\Delta - 2 \Big) \,, \tag{2.15}$$

where Δ represents the matter contribution to $\bar{L}_0$ other than the contribution due to the zero modes of $\bar{J}^i$, $\bar{\partial} X^\mu$. (In a toroidal compactification Δ would represent the oscillator contribution to $\bar{L}_0$). We need to calculate the degeneracy d of such states for large Δ. This can be done following the procedure developed in [65, 66, 67, 22] and the answer is:

$$d \simeq \exp \Big(2\pi \sqrt{\frac{c_L \Delta}{6}} \Big) \,, \tag{2.16}$$

where c_L is the total central charge of the left-handed part of the conformal field theory. If we work in the light-cone gauge so that all states are physical, we have

$$c_L = 24 \,. \tag{2.17}$$

This gives the following expression for the microscopic entropy:

$$S_{micro} \equiv \ln d \simeq 4\pi\sqrt{\Delta} = \frac{8\pi}{g} \sqrt{m^2 - \frac{Q_L^2}{8g^2}} \,, \tag{2.18}$$

[4] This condition is modified by string loop corrections, but we shall work at weak coupling where this relation is valid.

where we have used eq.(2.15) and the approximation $\Delta >> 1$. This is again identical to the expression for the microscopic entropy in heterotic string theory on T^6. (2.18) and (2.14) agree if

$$C = 4\,. \tag{2.19}$$

This is the same condition found in [12] for the agreement between the Bekenstein-Hawking entropy and the microscopic entropy for heterotic string theory on T^6. Thus we see that the criteria that guarantees the agreement between microscopic and the Bekenstein-Hawking entropy for electrically charged black holes in heterotic string theory on T^6 also guarantees similar agreement for $N = 2$ supersymmetric compactification of the heterotic string theory.

From this discussion it is clear that the same analysis can also be extended to more general $N = 4$ supersymmetric compactifications of the heterotic string theory described in [51]. The relevant part of the tree level effective action in these theories will be given by an appropriate restriction of the tree level effective action of heterotic string theory on T^6. Thus the Bekenstein-Hawking entropy of these black holes will again be given by (2.14) with the same constant C. Furthermore the microscopic entropy computed from the degeneracy of BPS states with a given set of charges is also given by the same formula (2.18). Thus demanding the agreement between the microscopic entropy and the Bekenstein-Hawking entropy gives us back the same equation (2.19).

It is now also clear how to extend this analysis to type II compactification. In this case the 'parent' theory will be type II string theory on T^6. Since the elementary string states carry only NS sector charges, we can restrict our attention to the effective action involving NS sector bosonic fields only. The low energy effective action is given by (2.5) with M representing an $O(6,6)$ matrix. There are higher derivative corrections to this action at the string tree level. These corrections are different from those in the heterotic string theory, since in this case there is local world-sheet supersymmetry in both the left and the right sector, and hence the vertex operators are constructed from superfields in both these sectors. This would modify the answer for the Bekenstein-Hawking entropy for an electrically chaged black hole to[5]

$$S_{BH} = \frac{2\pi C'}{g}\sqrt{m^2 - \frac{Q_L^2}{8g^2}}\,, \tag{2.20}$$

where C' is a new numerical constant. This is just as well, since the computation of the microscopic entropy in this theory also differs from that in the heterotic string theory. In particular, c_L appearing in eq.(2.16) now takes the value

$$c_L = 12\,, \tag{2.21}$$

[5] Here we are considering states which break all space-time supersymmetries coming from the left-moving sector of the world-sheet, and half of the supersymmetries coming from the right-moving sector.

JHEP02(1998)011

reflecting the contribution from the eight bosonic and eight fermionic world-sheet fields in the light-cone gauge. This gives

$$S_{micro} = \frac{4\sqrt{2}\pi}{g}\sqrt{m^2 - \frac{Q_L^2}{8g^2}}\,. \tag{2.22}$$

Agreement between (2.20) and (2.22) now requires

$$C' = 2\sqrt{2}\,. \tag{2.23}$$

Consider now any other compactification of the type II theory. Following the same argument as in the heterotic case, we see that the full tree level effective action involving the metric, and the NS sector vectors and scalars constructed from the $U(1)$ super-currents will be given by a restriction of the tree level effective action of type IIB on T^6. Thus the Bekenstein-Hawking entropy of an electrically charged BPS black hole will be given by an expression identical to (2.20), with the same constant C'. On the other hand the computation of the microscopic entropy also proceeds in an identical manner, with c_L given by 12 in each case. Thus the same equation (2.23) guarantees the agreement between the Bekenstein-Hawking entropy and the microscopic entropy in all cases.

3 Conclusion

In this paper we have generalised the analysis of refs. [12, 13] to black holes in more general heterotic and type II string theories in four dimensions with at least $N = 2$ supersymmetry. We have shown that in each case, the black hole entropy calculated from the string modified area of the event horizon agrees with the microscopic entropy calculated from the degeneracy of elementary string states, up to an overall numerical factor. Furthermore, we find that this numerical constant is universal in all heterotic string compactifications, and also in all type II compactifications. This means that if this constant has the correct value in one heterotic compactification, then it will automatically have the correct value for all heterotic string compactification. A similar result holds for type II compactification.

The explicit computation of this constant remains a problem for the future. The relevant σ-models describing the conformal field theory near the event horizon are the chiral null models discussed in [68]. But in order to compute the coefficient from these chiral null models one needs an abstract definition of the area of the event horizon in terms of the two dimensional conformal field theory describing string propagation in a black hole background. Such a definition is lacking at this moment.

In particular, there is a specific choice of metric such that the area of the event horizon, measured in this metric, remains zero to all orders in the σ-model loop expansion [68]. But there is no reason to believe that this is the metric that we should be using in computing the area of the event horizon while calculating the entropy. On the other hand, if we insist on working with this metric, then there is no reason to believe that the relationship between the entropy and the area of the event horizon does not get modified by higher loop terms in the σ-model.

References

[1] S. Hawking, *Mon. Not. R. Astron. Soc.* **152** (1991) 75;
A. Salam, in *Quantum Gravity: An Oxford Symposium*, eds. C. Isham, R. Penrose and D.W. Sciama (Oxford Univ. Press, 1975);
G. 't Hooft, *Nucl. Phys.* **B 335** (1990) 138.

[2] L. Susskind, `hep-th/9309145`;
L. Susskind and J. Uglam, *Phys. Rev.* **D 50** (1994) 2700 [`hep-th/9401070`];
J. Russo and L. Susskind, *Nucl. Phys.* **B 437** (1995) 611 [`hep-th/9405117`].

[3] M. Duff, R. Khuri, R. Minasian and J. Rahmfeld, *Nucl. Phys.* **B 418** (1994) 195 [`hep-th/9311120`];
M. Duff and J. Rahmfeld, *Phys. Lett.* **B 345** (1995) 441 [`hep-th/9406105`].

[4] C. Hull and P. Townsend, *Nucl. Phys.* **B 438** (1995) 109 [`hep-th/9410167`];
E. Witten, *Nucl. Phys.* **B 443** (1995) 85 [`hep-th/9503124`];
A. Strominger, *Nucl. Phys.* **B 451** (1995) 96 [`hep-th/9504090`];
B. Greene, D. Morrison and A. Strominger, *Nucl. Phys.* **B 451** (1995) 109 [`hep-th/9504145`].

[5] G. Horowitz and J. Polchinski, `hep-th/9612146`.

[6] E. Halyo, B. Kol, A. Rajaraman and L. Susskind, *Phys. Lett.* **B 401** (1997) 15 [`hep-th/9609075`].

[7] S. Mathur, `hep-th/9706151`.

[8] D. Youm, *Nucl. Phys.* **B 509** (1998) 431 [`hep-th/9706046`].

[9] S. Das, S. Kalyanarama, P. Ramadevi and S. Mathur, `hep-th/9711003`.

[10] K. Sfetsos and K. Skenderis, `hep-th/9711138`.

[11] E. Witten and D. Olive, *Phys. Lett.* **B 78** (1978) 97.

[12] A. Sen, *Mod. Phys. Lett.* **A 10** (1995) 2081 [`hep-th/9504147`].

[13] A. Peet, *Nucl. Phys.* **B 456** (1995) 732 [`hep-th/9506200`].

[14] R. Khuri and R. Myers, *Phys. Rev.* **D 52** (1995) 6988 [`hep-th/9508045`].

[15] C. Callan, J. Maldecena and A. Peet, *Nucl. Phys.* **B 475** (1996) 645 [`hep-th/9510134`].

[16] G. Mandal and S. Wadia, *Phys. Lett.* **B 372** (1996) 34 [hep-th/9511218].

[17] J. David, A. Dhar, G. Mandal and S. Wadia, *Phys. Lett.* **B 392** (1997) 39 [hep-th/9610120].

[18] R. Emparan, hep-th/9706204.

[19] A. Dabholkar, J. Gauntlett, J. Harvey and D. Waldram, *Nucl. Phys.* **B 474** (1996) 85 [hep-th/9511053].

[20] K. Suzuki, *Phys. Rev.* **D 56** (1997) 7846 [hep-th/9611095].

[21] J. Polchinski, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017], and references therein.

[22] A. Strominger and C. Vafa, *Phys. Lett.* **B 379** (1996) 99 [hep-th/9601029].

[23] For a review, see J. Maldecena, *Nucl. Phys.* **B 61** *(Proc. Suppl.)* (1998) 111 [hep-th/9705078].

[24] S. Ferrara, R. Kallosh and A. Strominger, *Phys. Rev.* **D 52** (1995) 5412 [hep-th/9508072].

[25] A. Strominger, *Phys. Lett.* **B 383** (1996) 39 [hep-th/9602111].

[26] D. Kaplan, D. Lowe, J. Maldacena and A. Strominger, *Phys. Rev.* **D 55** (1997) 4898 [hep-th/9609204].

[27] J. Maldacena, A. Strominger and E. Witten, **JHEP 12** (1997) 002 [hep-th/9711053].

[28] C. Vafa, hep-th/9711067.

[29] S. Ferrara and R. Kallosh, *Phys. Rev.* **D 54** (1996) 1514 [hep-th/9602136]; *Phys. Rev.* **D 54** (1996) 1525 [hep-th/9603090].

[30] M. Shmakova, *Phys. Rev.* **D 56** (1997) 540 [hep-th/9612076].

[31] K. Behrndt, G. Lopez Cardoso, B. de Wit, R. Kallosh, D. Lust and T. Mohaupt, *Nucl. Phys.* **B 488** (1997) 236 [hep-th/9610105].

[32] K. Behrndt and T. Mohaupt, *Phys. Rev.* **D 56** (1997) 2206 [hep-th/9611140].

[33] J. Maldacena, *Phys. Lett.* **B 403** (1997) 20 [hep-th/9611163].

[34] C. Callan and J. Maldacena, *Nucl. Phys.* **B 472** (1996) 591 [hep-th/9602043]; G. Horowitz and A. Strominger, *Phys. Rev. Lett.* **77** (1996) 2368 [hep-th/9602051].

[35] A. Dhar, G. Mandal and S. Wadia, *Phys. Lett.* **B 388** (1996) 51 [hep-th/9605234].

[36] S. Das and S. Mathur, *Nucl. Phys.* **B 478** (1996) 561 [hep-th/9606185].

[37] J. Maldacena and A. Strominger, *Phys. Rev.* **D 55** (1997) 861 [hep-th/9609026].

[38] F. Larsen and F. Wilczek, *Phys. Lett.* **B 375** (1996) 37 [hep-th/9511064]; *Nucl. Phys.* **B 475** (1996) 627 [hep-th/9604134]; *ibid.* **488** (1997) 261 [hep-th/9609084].

[39] M. Cvetic and A. Tseytlin, *Phys. Rev.* **D 53** (1996) 5619 [hep-th/9512031].

[40] A. Tseytlin, *Mod. Phys. Lett.* **A 11** (1996) 689 [hep-th/9601177]; *Nucl. Phys.* **B 477** (1996) 431 [hep-th/9605091].

[41] J. Maldecena, *Nucl. Phys.* **B 477** (1996) 168 [hep-th/9605016].

[42] M. Cvetic and D. Youm, *Nucl. Phys.* **B 477** (1996) 449 [hep-th/9605051].

[43] M. Duff, J. Liu and J. Rahmfeld, *Nucl. Phys.* **B 494** (1997) 161 [hep-th/9612015].

[44] A. Dabholkar, *Phys. Lett.* **B 402** (1997) 53 [hep-th/9702050].

[45] A. Dabholkar, G. Mandal and P. Ramadevi, hep-th/9705239.

[46] N. Seiberg and E. Witten, *Nucl. Phys.* **B 426** (1994) 19 [hep-th/9407087].

[47] K. Narain, *Phys. Lett.* **B 169** (1986) 41;
K. Narain, H. Sarmadi and E. Witten, *Nucl. Phys.* **B 279** (1987) 369.

[48] T. Banks, L. Dixon, D. Friedan and E. Martinec, *Nucl. Phys.* **B 299** (1988) 613.

[49] T. Banks and L. Dixon, *Nucl. Phys.* **B 307** (1988) 93.

[50] R. Blum, L. Dolan and P. Goddard, *Nucl. Phys.* **B 289** (1987) 364 ;
H. Kawai, D. Lewellen and H. Tye, *Phys. Lett.* **B 191** (1987) 63 ;
W. Lerche, B. Nilsson and A. Schellekens, *Nucl. Phys.* **B 294** (1987) 136 ;
S. Ferrara and C. Kounnas, *Nucl. Phys.* **B 328** (1989) 406 ;
L. Dixon, V. Kaplunovsky and C. Vafa, *Nucl. Phys.* **B 294** (1987) 43.

[51] S. Chaudhuri, G. Hockney and J. Lykken, *Phys. Rev. Lett.* **75** (1995) 2264 [hep-th/9505054].

[52] S. Ferrara, C. Kounnas and M. Porrati, *Phys. Lett.* **B 181** (1986) 263.

[53] M. Cvetic, J. Louis and B. Ovrut, *Phys. Lett.* **B 206** (1988) 227.

[54] G. Lopes Cardoso, D. Lust and T. Mohaupt, *Nucl. Phys.* **B 432** (1994) 68 [hep-th/940500].

[55] B. de Wit, V. Kaplunovsky, J. Louis and D. Lust, *Nucl. Phys.* **B 451** (1995) 53 [hep-th/9504006]

[56] I. Antoniadis, S. Ferrara, E. Gava, K. Narain and T. Taylor, *Nucl. Phys.* **B 447** (1995) 35 [hep-th/9504034].

[57] P. Fre, *Nucl. Phys.* **B 45BC** *(Proc. Suppl.)* (1996) 59 [hep-th/9512043] and references therein.

[58] A. Sen, *Int. J. Mod. Phys.* **A 9** (1994) 3707 [hep-th/9402002].

[59] J. Maharana and J. Schwarz, *Nucl. Phys.* **B 390** (1993) 3 [hep-th/9207016].

[60] A. Sen, *Phys. Rev.* **D 32** (1985) 2102; *Phys. Rev. Lett.* **55** (1985) 1846.

[61] C. Callan, D. Friedan, E. Martinec and M. Perry, *Nucl. Phys.* **B 262** (1985) 593.

[62] D. Friedan, E. Martinec and S. Shenker, *Nucl. Phys.* **B 271** (1986) 93; *Phys. Lett.* **B 160** (1985) 55.

[63] V. Knizhnik, *Phys. Lett.* **B 160** (1985) 403.

[64] J. Lauer, D. Lust and S. Theisen, *Nucl. Phys.* **B 309** (1988) 771.

[65] J. Cardy, *Nucl. Phys.* **B 270** (1986) 186.

[66] I. Kani and C. Vafa, *Comm. Math. Phys.* **130** (1990) 529.

[67] J. Harvey and G. Moore, *Nucl. Phys.* **B 463** (1996) 315 [hep-th/9510182].

[68] G. Horowitz and A. Tseytlin, *Phys. Rev. Lett.* **73** (1994) 3351 [hep-th/9408040].